Bell Bases Decomposition of a General N-qubit State Teleportation through a Non-maximally Entangled Quantum Channel

Xiu-Lao Tian, Xiao-Qiang Xi
Department of Applied Mathematics and Physics,
Xi'an Institute of Posts and Telecommunications, Xi'an 710061, China

(Dated: February 1, 2008)

We propose a method of the Bell bases decomposition to teleport an arbitrary unknown N-qubit state through a nonmaximally entangled quantum channel, and give the universal decomposition matrix of N-qubit. Using the decomposition matrix, we can easily obtain the collapsed state at the receiver site. The inverse matrix that the decomposition matrix is just the transformation matrix that the receiver can manipulate. The decomposition matrix is a function of the parameters of the quantum channel. After defining the sub-matrix of the quantum channel, we find that the decomposition matrix is a tensor product of the sub-matrixes.

PACS numbers: 03.67.Hk

1. INTRODUCTION

Quantum entanglement plays an important role in the field of quantum information processing, such as quantum computation, quantum cryptography, quantum teleportation, dense coding and so on [1]. Since Bennett *et al.* [2] presented a quantum teleportation scheme, there has been a great development of quantum communication in experiments [3, 4, 5, 6] and theory. The investigations of N (N$\geq$ 2) qubits teleportation have attracted much attention, such as, two-qubit teleportation [7] and probabilistic teleportation [8], teleportation of a special GHZ state [9], probabilistically teleport a three-particle state via three pairs of entangled particles [10]. Besides the discrete-variable systems, continuous-variable systems [11, 12] are also studied in teleportation.

There are three key problems and a difficult problem in the processing of teleportation. The key problems are: (1) the sender (Alice) and the receiver (Bob) must share the quantum channels; (2) Alice performs the Bell bases measurement on her particles; (3) Bob performs a corresponding transformation to his particles to reestablish the transmitted state with a certain probability. The difficult problem is multi-qubit teleportation. It is well known that the Bell bases measurement is equivalent to the joint Bell bases acting on the joint system, if we can decompose the joint system with Bell bases state, then we can easily obtain the results of the Bell bases measurement, it is meaningful to find the Bell bases decomposition of the joint system. In a practical teleportation, due to the effect of the environment and the channel noises, the quantum channel is not always in a maximally entangled state (MES). Therefore, it is very important to study the quantum teleportation of an unknown quantum state by using a nonmaximally entangled channel (NMES) [13, 14].

Recently, several researchers investigated N($\geq$ 2)-qubit teleportation, but they only use a special nonmaximally entangled quantum channel or teleport a special state. Here we use GENERAL in order to different from that of Ref. [15, 16, 17, 18], Certainly, it is very complex and difficult to study a GENERAL N($\geq$ 2)-qubit teleportation through a GENERAL NONMAXIMALLY (GGNM). In this paper, in order to solve this GGNM problems, we propose a method of the Bell bases decomposition in teleportation and give the universal decomposition matrix of N-qubit teleportation and maybe the tensor product of the sub-matrixes can open out some characteristic of teleportation. Using decomposition matrix, we can easily obtain some useful information about teleportation.

2. BELL BASES DECOMPOSITON OF A GENERAL THREE-QUBIT AND FOUR-QUBIT STATE TELEPORTATION

2.1 Three-qubit case

Supposing Alice wants to send an unknown general three-qubit state $|\varphi>_{1,2,3}$ to Bob,

$$\begin{aligned} |\varphi>_{1,2,3} &= \sum_{i=1}^{2^3} X_i|x_i^1 x_i^2 x_i^3> \\ &= X_1|000>+X_2|001>+X_3|010>+X_4|011>+X_5|100>+X_6|101>+X_7|110>+X_8|111> . \quad (1) \end{aligned}$$

Using three general entangled pairs as the quantum channel, they are

$$|\varphi>_{4,5} = \sum_{j^1=1}^{4} Y_j^1|y_j^1 y_j^{'1}> = Y_1^1|00> + Y_2^1|01> + Y_3^1|10> + Y_4^1|11>, \tag{2}$$

$$|\varphi>_{6,7} = \sum_{j^2=1}^{4} Y_j^2|y_j^2 y_j^{'2}> = Y_1^2|00> + Y_2^2|01> + Y_3^2|10> + Y_4^2|11>, \tag{3}$$

$$|\varphi>_{8,9} = \sum_{j^3=1}^{4} Y_j^2|y_j^3 y_j^{'3}> = Y_1^3|00> + Y_2^3|01> + Y_3^3|10> + Y_4^3|11> . \tag{4}$$

Where $j^i(i=1,2,3)$ denotes the i-th entangled pair of the quantum channel, similarly later. The state of the joint system can be written as

$$|\Psi_T> = |\varphi_{1,2,3} \otimes \varphi_{4,5} \otimes \varphi_{6,7} \otimes \varphi_{8,9}> = \sum_{i=1}^{4}\sum_{j^1=1}^{4}\sum_{j^2=1}^{4}\sum_{j^3=1}^{4} X_i Y_j^1 Y_j^2 Y_j^3 |x_i^1 x_i^2 x_i^3 y_j^1 y_j^{'1} y_j^2 y_j^{'2} y_j^3 y_j^{'3}>_{1-9}, \tag{5}$$

where $X_i Y_j^1 Y_j^2 Y_j^3$ is the coefficient, $|x_i^1 x_i^2 x_i^3>$ instead of the micro state of three particle and $|y_j^i y_j^{'i}>$ $(i=1,2,3)$ instead of the micro state of two particle. The value of the indexes in different qubits can be seen from Eq.(1)-(4), they are $x_{1,2,3,4}^1 = x_{1,2,5,6}^2 = x_{1,3,5,7}^3 = 0$, $x_{5,6,7,8}^1 = x_{3,4,7,9}^2 = x_{2,4,6,8}^3 = 1$, $y_{1,2}^1 = y_{1,3}^{'1} = y_{1,2}^2 = y_{1,3}^{'2} = y_{1,2}^3 = y_{1,3}^{'3} = 0$, $y_{3,4}^1 = y_{2,4}^{'1} = y_{3,4}^2 = y_{2,4}^{'2} = y_{3,4}^3 = y_{2,4}^{'3} = 1$. Let particles 1,2,3,5,7,9 belong to Alice and 4,6,8 belong to Bob, Alice can measure $|\Psi_T>$ with joint Bell bases $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}>$, where $\varphi_{i,j}^k$ is the Bell bases $|\varphi_{i,j}^1> = (|00>+|11>)/\sqrt{2}, |\varphi_{i,j}^2> = (|00>-|11>)/\sqrt{2}, |\varphi_{i,j}^3> (|01>+|10>)/\sqrt{2}, |\varphi_{i,j}^4> = (|01>-|10>)/\sqrt{2}$.

So we can rearrange the qubits in $|\Psi_T>$ by the order of 1-5-2-7-3-9-4-6-8 and rewrite $|\Psi_T>$ as

$$|\Psi_T> = \sum_{i=1}^{4}\sum_{j^1=1}^{4}\sum_{j^2=1}^{4}\sum_{j^3=1}^{4} X_i Y_j^1 Y_j^2 Y_j^3 |(x_i^1 y_j^{'1} x_i^2 y_j^{'2} x_i^3 y_j^{'3})_{1,5,2,7,3,9}(y_j^1 y_j^2 y_j^3)_{4,6,8}> . \tag{6}$$

Under the bases of three qubit $(|000>, |001>, |010>, |011>, |100>, |101>, |110>, |111>)$, the matrix expression of $|\Psi_T>$ by the Bell bases decomposition of $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}>$ is

$$|\Psi_T> = (1/\sqrt{2})^3 \sum_{\alpha_1=1}^{4}\sum_{\alpha_2=1}^{4}\sum_{\alpha_3=1}^{4} \varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}|\varphi_{4,6,8}>, \tag{7}$$

where $|\varphi_{4,6,8}> = (X_1, X_2, \ldots, X_8)_{4,6,8}^T$, $\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}$ is called the decomposition matrix. Alice measures $|\Psi_T>$ with the joint Bell bases of $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}>$ is equivalent to the action of $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}>$ on $|\psi_T>$, and Bob gets the possible collapsed quantum state $(1/\sqrt{2})^3\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}|\varphi_{4,6,8}>$, i.e.

$$<\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}|\Psi_T> = (1/\sqrt{2})^3\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}|\varphi_{4,6,8}> . \tag{8}$$

We can calculate the decomposition matrixes $\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}|\varphi_{4,6,8}>$ through Eq.(8) and induce its general expressions,

$$|\varphi_{1,5}^1\varphi_{2,7}^1\varphi_{3,9}^1> = (1/\sqrt{2})^3(|00>+|11>)_{1,5}(|00>+|11>)_{2,7}(|00>+|11>)_{3,9}. \tag{9}$$

From the value of the indexes in different qubits, we can get the micro states in Eq.(9), see Table 1.

Table 1. The states under general bases and the joint Bell bases $|\varphi_{1,5}^1\varphi_{2,7}^1\varphi_{3,9}^1>$.

state	state under joint Bell bases	state	state under joint Bell bases
$\|000000>$	$\sum_{j^1=1,3}\sum_{j^2=1,3}\sum_{j^3=1,3} X_1 Y_j^1 Y_j^2 Y_j^3 \|x_1^1 y_j^{'1} x_1^2 y_j^{'2} x_1^3 y_j^{'3}>$	$\|000011>$	$\sum_{j^1=1,3}\sum_{j^2=1,3}\sum_{j^3=2,4} X_2^2 Y_j^1 Y_j^2 Y_j^3 \|x_2^1 y_j^{'1} x_2^2 y_j^{'2} x_2^3 y_j^{'3}>$
$\|001100>$	$\sum_{j^1=1,3}\sum_{j^2=2,4}\sum_{j^3=1,3} X_3 Y_j^1 Y_j^2 Y_j^3 \|x_3^1 y_j^{'1} x_3^2 y_j^{'2} x_3^3 y_j^{'3}>$	$\|001111>$	$\sum_{j^1=1,3}\sum_{j^2=2,4}\sum_{j^3=2,4} X_4 Y_j^1 Y_j^2 Y_j^3 \|x_4^1 y_j^{'1} x_4^2 y_j^{'2} x_4^3 y_j^{'3}>$
$\|110000>$	$\sum_{j^1=2,4}\sum_{j^2=1,3}\sum_{j^3=1,3} X_5 Y_j^1 Y_j^2 Y_j^3 \|x_5^1 y_j^{'1} x_5^2 y_j^{'2} x_5^3 y_j^{'3}>$	$\|110011>$	$\sum_{j^1=2,4}\sum_{j^2=1,3}\sum_{j^3=2,4} X_6 Y_j^1 Y_j^2 Y_j^3 \|x_6^1 y_j^{'1} x_6^2 y_j^{'2} x_6^3 y_j^{'3}>$
$\|111100>$	$\sum_{j^1=2,4}\sum_{j^2=2,4}\sum_{j^3=1,3} X_7 Y_j^1 Y_j^2 Y_j^3 \|x_7^1 y_j^{'1} x_7^2 y_j^{'2} x_7^3 y_j^{'3}>$	$\|111111>$	$\sum_{j^1=2,4}\sum_{j^2=2,4}\sum_{j^3=2,4} X_8 Y_j^1 Y_j^2 Y_j^3 \|x_8^1 y_j^{'1} x_8^2 y_j^{'2} x_8^3 y_j^{'3}>$

$\sigma^{111}_{4,6,8}|\varphi_{4,6,8}>$ can be written as

$$\begin{aligned}\sigma^{111}_{4,6,8}|\varphi_{4,6,8}> &= [X_1\sum_{j^1=1,3}\sum_{j^2=1,3}\sum_{j^3=1,3}+X_2\sum_{j^1=1,3}\sum_{j^2=1,3}\sum_{j^3=2,4}+X_3\sum_{j^1=1,3}\sum_{j^2=2,4}\sum_{j^3=1,3}\\ &+ X_4\sum_{j^1=1,3}\sum_{j^2=2,4}\sum_{j^3=2,4}+X_5\sum_{j^1=2,4}\sum_{j^2=1,3}\sum_{j^3=1,3}+X_6\sum_{j^1=2,4}\sum_{j^2=1,3}\sum_{j^3=2,4}\\ &+ X_7\sum_{j^1=2,4}\sum_{j^2=2,4}\sum_{j^3=1,3}+X_8\sum_{j^1=2,4}\sum_{j^2=2,4}\sum_{j^3=2,4}]Y^1_jY^2_jY^3_j|y^1_jy^2_jy^3_j>_{4,6,8}\,. \end{aligned} \tag{10}$$

Through tedious calculation and using $Y^1_jY^2_jY^3_j$ instead of $Y^1_jY^2_jY^3_j|y^1_jy^2_jy^3_j>$ we get

$$\begin{aligned}\sigma^{111}_{4,6,8}|\varphi_{4,6,8}> &= X_1(Y^1_1+Y^1_3)(Y^2_1+Y^2_3)(Y^3_1+Y^3_3)+X_2(Y^1_1+Y^1_3)(Y^2_1+Y^2_3)(Y^3_2+Y^3_4)\\ &+ X_3(Y^1_1+Y^1_3)(Y^2_2+Y^2_4)(Y^3_1+Y^3_3)+X_4(Y^1_1+Y^1_3)(Y^2_2+Y^2_4)(Y^3_2+Y^3_4)\\ &+ X_5(Y^1_2+Y^1_4)(Y^2_1+Y^2_3)(Y^3_1+Y^3_3)+X_6(Y^1_2+Y^1_4)(Y^2_1+Y^2_3)(Y^3_2+Y^3_4)\\ &+ X_7(Y^1_2+Y^1_4)(Y^2_2+Y^2_4)(Y^3_1+Y^3_3)+X_8(Y^1_2+Y^1_4)(Y^2_2+Y^2_4)(Y^3_2+Y^3_4). \end{aligned} \tag{11}$$

In the bases of $(X_1,X_2,\ldots,X_8)^T_{4,6,8}$, $\sigma^{111}_{4,6,8}$ can be written as the tensor product:

$$\sigma^{111}_{4,6,8}=\begin{pmatrix}Y^1_1 & Y^1_2\\ Y^1_3 & Y^1_4\end{pmatrix}\otimes\begin{pmatrix}Y^2_1 & Y^2_2\\ Y^2_3 & Y^2_4\end{pmatrix}\otimes\begin{pmatrix}Y^3_1 & Y^3_2\\ Y^3_3 & Y^3_4\end{pmatrix}. \tag{12}$$

Similarly, we can obtain

$$\sigma^{234}_{4,6,8}=\begin{pmatrix}Y^1_1 & -Y^1_2\\ Y^1_3 & -Y^1_4\end{pmatrix}\otimes\begin{pmatrix}Y^2_2 & Y^2_1\\ Y^2_4 & Y^2_3\end{pmatrix}\otimes\begin{pmatrix}Y^3_2 & -Y^3_1\\ Y^3_4 & -Y^3_3\end{pmatrix}. \tag{13}$$

Through analysis and validation we find that $\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8}$ has an general expression

$$\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8}=\sigma^{\alpha_1}_4\otimes\sigma^{\alpha_2}_6\otimes\sigma^{\alpha_3}_8, \tag{14}$$

where $\alpha_1,\alpha_2,\alpha_3=1,2,3,4$ and $\sigma^{\alpha_1}_4,\sigma^{\alpha_2}_6,\sigma^{\alpha_3}_8$ are called the sub-matrixes of the decomposition matrix $\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8}$, the expression of them are shown in Table 2.

Table 2. The sub-matrixes of the three-qubit decomposition matrix, $i=1,2,3$ instead of the i-th entangled pair.

σ^1	σ^2	σ^3	σ^4
$\begin{pmatrix}Y^i_1 & Y^i_2\\ Y^i_3 & Y^i_4\end{pmatrix}$	$\begin{pmatrix}Y^i_1 & -Y^i_2\\ Y^i_3 & -Y^i_4\end{pmatrix}$	$\begin{pmatrix}Y^i_2 & Y^i_1\\ Y^i_4 & Y^i_3\end{pmatrix}$	$\begin{pmatrix}Y^i_2 & -Y^i_1\\ Y^i_4 & -Y^i_3\end{pmatrix}$

$\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8}$ is a function of the parameters of the quantum channel′ , different decomposition matrix corresponds to the different quantum channels.

Using the method of tensor product, we find that the sub-matrixes have a universal expression

$$\sigma^{\mu}_i=\begin{pmatrix}T^i_{\mu1}Y^i_1+T^i_{\mu2}Y^i_2 & T^i_{\mu3}Y^i_1+T^i_{\mu4}Y^i_2\\ T^i_{\mu1}Y^i_3+T^i_{\mu2}Y^i_4 & T^i_{\mu3}Y^i_3+T^i_{\mu4}Y^i_4\end{pmatrix}, \tag{15}$$

where $\mu=1,2,3,4$, $i=1,2,3$ and $T_{\mu k}(k=1,2,3,4)$ is the elements of the transformation matrix between the Bell bases and the general bases of two-qubit $(|00>,|01>,|10>,|11>)$

$$\begin{pmatrix}\varphi^1_{i,j}\\ \varphi^2_{i,j}\\ \varphi^3_{i,j}\\ \varphi^4_{i,j}\end{pmatrix}=\frac{1}{\sqrt{2}}T_{ij}\begin{pmatrix}00\\01\\10\\11\end{pmatrix}_{ij}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\1&0&0&-1\\0&1&1&0\\0&1&-1&0\end{pmatrix}_{ij}\begin{pmatrix}00\\01\\10\\11\end{pmatrix}_{ij}. \tag{16}$$

2.2 Four-qubit case

Supposing Alice wants to send an unknown general four-qubit state $|\varphi>_{1,2,3,4}$ to Bob,

$$\begin{aligned} |\varphi>_{1,2,3,4} &= \sum_{i=1}^{2^4} X_i|x_i^1x_i^2x_i^3x_i^4> \\ &= X_1|0000>+X_2|0001>+X_3|0010>+X_4|0011>+X_5|0100>+X_6|0101> \\ &\quad +X_7|0110>+X_8|0111>+X_9|1000>+X_{10}|1001>+X_{11}|1010>+X_{12}|1011> \\ &\quad +X_{13}|1100>+X_{14}|1101>+X_{15}|1110>+X_{16}|1111>. \end{aligned} \tag{17}$$

Using four entangled pairs as the quantum channel, they are

$$|\varphi>_{5,6} = \sum_{j^1=1}^{4} Y_j^1|y_j^1y_j^{'1}>_{5,6}= Y_1^1|00>+Y_2^1|01>+Y_3^1|10>+Y_4^1|11>, \tag{18}$$

$$|\varphi>_{7,8} = \sum_{j^2=1}^{4} Y_j^2|y_j^1y_j^{'2}>_{7,8}= Y_1^2|00>+Y_2^2|01>+Y_3^2|10>+Y_4^2|11>, \tag{19}$$

$$|\varphi>_{9,10} = \sum_{j^3=1}^{4} Y_j^3|y_j^1y_j^{'3}>_{9,10}= Y_1^3|00>+Y_2^3|01>+Y_3^3|10>+Y_4^3|11>, \tag{20}$$

$$|\varphi>_{11,12} = \sum_{j^4=1}^{4} Y_j^4|y_j^1y_j^{'4}>_{11,12}= Y_1^4|00>+Y_2^4|01>+Y_3^4|10>+Y_4^4|11>. \tag{21}$$

The state of the joint system can be written as

$$|\Psi_T>=|\varphi_{1,2,3,4}\otimes\varphi_{5,6}\otimes\varphi_{7,8}\otimes\varphi_{9,10}\otimes\varphi_{11,12}>=\sum_{i}^{4}\sum_{j^1}^{4}\sum_{j^2}^{4}\sum_{j^3}^{4}\sum_{j^4}^{4} X_iY_j^1Y_j^2Y_j^3Y_j^4|x_i^1x_i^2x_i^3x_i^4y_j^1y_j^{'1}y_j^2y_j^{'2}y_j^3y_j^{'3}y_j^4y_j^{'4}>. \tag{22}$$

Let particles 1,2,3,4,5,7,9,11 belong to Alice and 6,8,10,12 belong to Bob, then Alice can measure $|\Psi_T>$ with joint Bell bases $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}\varphi_{4,11}^{\alpha_4}>$.

So we can rearrange the qubits in $|\Psi_T>$ by the order of 1-5-2-7-3-9-4-11-6-8-10-12 and rewrite $|\Psi_T>$ as

$$|\Psi_T>=\sum_{i=1}^{4}\sum_{j^1=1}^{4}\sum_{j^2=1}^{4}\sum_{j^3=1}^{4}\sum_{j^4=1}^{4} X_iY_j^1Y_j^2Y_j^3Y_j^4|(x_i^1y_j^1x_i^2y_j^2x_i^3y_j^3x_i^4y_j^4)_{1,5,2,7,3,9,4,11}(y_j^{'1}y_j^{'2}y_j^{'3}y_j^{'4})_{6,8,10,12}>. \tag{23}$$

Under the bases of four qubit, the matrix expression of $|\Psi_T>$ by the Bell bases decomposition of $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}\varphi_{4,11}^{\alpha_4}>$ is

$$|\Psi_T>=(1/\sqrt{2})^4\sum_{\alpha_1=1}^{4}\sum_{\alpha_2=1}^{4}\sum_{\alpha_3=1}^{4}\sum_{\alpha_4=1}^{4}\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}\varphi_{4,11}^{\alpha_4}\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3\alpha_4}|\varphi_{6,8,10,12}>, \tag{24}$$

where $|\varphi>_{6,8,10,12}=(X_1,X_2,\ldots,X_{16})^T_{6,8,10,12}$, $\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3}$ is the decomposition matrix of four-qubit state teleportation. Alice measures $|\Psi_T>$ with the joint Bell bases of $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}\varphi_{4,11}^{\alpha_4}>$, the collapsed quantum state that Bob can obtain is $|(1/\sqrt{2})^4\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3\alpha_4}|\varphi_{6,8,10,12}>$, i.e.

$$<\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}\varphi_{4,11}^{\alpha_3}|\Psi_T>=(1/\sqrt{2})^4\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3\alpha_4}|\varphi_{6,8,10,12}>. \tag{25}$$

The general expression of $\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3\alpha_4}$ is

$$\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3\alpha_4}=\sigma_6^{\alpha_1}\otimes\sigma_8^{\alpha_2}\otimes\sigma_{10}^{\alpha_3}\otimes\sigma_{12}^{\alpha_4}, \tag{26}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 = 1, 2, 3, 4$ and $\sigma_6^{\alpha_1}$, $\sigma_8^{\alpha_2}, \sigma_{10}^{\alpha_3}, \sigma_{12}^{\alpha_4}$ are the sub-matrixes,the expression of them are shown in Table 3.

Table 3. The sub-matrixes of the four-qubit decomposition matrix, $i = 1, 2, 3, 4$ instead of the i-th entangled pair.

σ^1	σ^2	σ^3	σ^4
$\begin{pmatrix} Y_1^i & Y_3^i \\ Y_2^i & Y_4^i \end{pmatrix}$	$\begin{pmatrix} Y_1^i & -Y_3^i \\ Y_2^i & -Y_4^i \end{pmatrix}$	$\begin{pmatrix} Y_3^i & Y_1^i \\ Y_4^i & Y_2^i \end{pmatrix}$	$\begin{pmatrix} Y_3^i & -Y_1^i \\ Y_4^i & -Y_2^i \end{pmatrix}$

$\sigma_{6,8,10,12}^{\alpha_1\alpha_2\alpha_3\alpha_4}$ is the function of the parameters of the quantum channel, different quantum channel corresponds to differences decomposition matrix. There have been some different between Table 3 and Table 2, because we have chosen different Bell bases in three qubit and four qubit cases.

3. BELL BASES DECOMPOSITION OF A GENERAL N-QUBIT STATE TELEPORTATION

Supposing Alice wants to send an unknown N-qubit state $|\varphi>_{1,2,\ldots,N}$ to Bob,

$$|\varphi>_{1,2,\ldots,N}=\sum_{i=1}^{2^N} X_i|x_i^1 x_i^2 \ldots x_i^N> . \tag{27}$$

Using N entangled pairs as the quantum channel, they are

$$|\varphi>_{N+(2n-1),N+2n}=\sum_{j=1}^{4} Y_j^n|y_j^n y_j^{'n}>= Y_1^n|00>+Y_2^n|01>+Y_3^n|10>+Y_4^n|11>, \tag{28}$$

where $n = 1, 2, \ldots, N$.

The state of the joint system can be written as

$$\begin{aligned} |\Psi_T> &= \varphi_{1,2,\ldots,N} \otimes \varphi_{N+1,N+2} \otimes \cdots \otimes \varphi_{N+(2N-1),N+2N} \\ &= \sum_i^4 \underbrace{\sum_{j^1}^4 \cdots \sum_{j^N}^4}_{N} X_i Y_j^1 Y_j^2 \ldots Y_j^N |x_i^1 x_i^2 \ldots x_i^N y_j^1 y_j^{'1} \ldots y_j^N y_j^{'N}>_{1-3N}, \end{aligned} \tag{29}$$

Let particle $1, 2, \ldots, N, N+1, N+3, \ldots, N+(2N-1)$ belong to Alice and $N+2, N+4, \ldots, N+2N$ belong to Bob, then Alice can measure $|\Psi_T>$ with Bell bases $|\varphi_{1,N+1}^{\alpha_1}\varphi_{2,N+3}^{\alpha_2}\cdots\varphi_{N,N+(2N-1)}^{\alpha_N}>$ $(\alpha_{1,2,\ldots,N} = 1, 2, 3, 4)$, where $\varphi_{i,j}^k$ is the Bell bases $|\varphi_{i,j}^1>= (|00>+|11>)/\sqrt{2}, |\varphi_{i,j}^2>= (|00>-|11>)/\sqrt{2}, |\varphi_{i,j}^3>= (|01>+|10>)/\sqrt{2}, |\varphi_{i,j}^4>= (|01>-|10>)/\sqrt{2}$.

We can rearrange the order of the particles as $1-(N+1)-2-(N+3)-\cdots-N-(N+2N-1)-(N+2)-(N+4)-\cdots-(N+2N)$ and rewrite $|\Psi_T>$ as

$$|\Psi_T>=\sum_i^4 \underbrace{\sum_{j^1}^4 \cdots \sum_{j^N}^4}_{N} X_i Y_j^1 Y_j^2 \ldots Y_j^N |(x_i^1 y_j^1 x_i^2 y_j^2 \ldots x_i^N y_j^N)(y_j^{'1} \ldots y_j^{'N})>_{1-3N} . \tag{30}$$

In the bases of N qubit, the matrix expression of $|\Psi_T>$ by the Bell bases decomposition of $|\varphi_{1,N+1}^{\alpha_1}\varphi_{2,N+3}^{\alpha_2}\cdots\varphi_{N,N+(2N-1)}^{\alpha_N}>$ is

$$|\Psi_T>= (1/\sqrt{2})^N \sum_{\alpha_1=1}^4 \sum_{\alpha_2=1}^4 \cdots \sum_{\alpha_N=1}^4 \varphi_{1,N+1}^{\alpha_1}\varphi_{2,N+3}^{\alpha_2}\cdots\varphi_{N,N+(2N-1)}^{\alpha_N}\sigma_{N+2,N+4,\ldots,N+2N}^{\alpha_1\ldots\alpha_N}|\varphi_{N+2,N+4,\ldots,N+2N}> \tag{31}$$

where $|\varphi_{N+2,N+4,\ldots,N+2N}>=(X_1,X_2,\ldots,X_N)^T_{N+2,N+4,\ldots,N+2N}$, and $\sigma^{\alpha_1\ldots\alpha_N}_{N+2,N+4,\ldots,N+2N}$ is the decomposition matrix. Alice measure $|\Psi_T>$ with the Bell bases of $|\varphi^{\alpha_1}_{1,N+1}\varphi^{\alpha_2}_{2,N+3}\cdots\varphi^{\alpha_N}_{N,N+(2N-1)}>$, the collapsed quantum state that Bob can obtain is $(1/\sqrt{2})^N\sigma^{\alpha_1\ldots\alpha_N}_{N+2,N+4,\ldots,N+2N}\varphi_{N+2,N+4,\ldots,N+2N}$, i.e.

$$<\varphi^{\alpha_1}_{1,N+1}\varphi^{\alpha_2}_{2,N+3}\cdots\varphi^{\alpha_N}_{N,N+(2N-1)}|\Psi_T>=(1/\sqrt{2})^N\sigma^{\alpha_1\ldots\alpha_N}_{N+2,N+4,\ldots,N+2N}|\varphi_{N+2,N+4,\ldots,N+2N}>. \tag{32}$$

$\sigma^{\alpha_1\ldots\alpha_N}_{N+2,N+4,\ldots,N+2N}$ has the form of

$$\sigma^{\alpha_1\ldots\alpha_N}_{N+2,N+4,\ldots,N+2N}=\sigma^{\alpha_1}_{N+2}\otimes\sigma^{\alpha_2}_{N+4}\otimes\cdots\otimes\sigma^{\alpha_N}_{N+2N}, \tag{33}$$

Using the method of tensor product, we find that the sub-matrixes have a simple universal expression

$$\sigma^\mu_i=\begin{pmatrix} T^i_{\mu1}Y^i_1+T^i_{\mu2}Y^i_3 & T^i_{\mu3}Y^i_1+T^i_{\mu4}Y^i_3 \\ T^i_{\mu1}Y^i_2+T^i_{\mu2}Y^i_4 & T^i_{\mu3}Y^i_2+T^i_{\mu4}Y^i_4 \end{pmatrix}, \tag{34}$$

where $\mu=1,2,3,4$, $i=1,2,\ldots,N$, the odd N, the even N and the choice of Bell bases will affect the expression of σ^μ_i.

4. THE APPLICATIONS OF THE DECOMPOSITION MATRIX

This method of the Bell bases decomposition for arbitrary qubit teleportation is universal.for example , Supposing Alice wants to send an unknown general single-qubit state $|\varphi>_1$ to Bob,

$$|\varphi>_1=\sum_{i=1}^{2}X_i|x^i>=X_1|0>+X_2|1>. \tag{35}$$

the quantum channel is

$$|\varphi>_{2,3,}\;=\;\sum_{j^1=1}^{4}Y^1_j|y^1_jy'^1_j>=Y^1_1|00>+Y^1_2|01>+Y^1_3|10>+Y^1_4|11> \tag{36}$$

the joint system is

$$|\Psi_T>_{1,2,3,}=\varphi_1\otimes\varphi_{2,3} \tag{37}$$

using Bell bases $|\varphi^\alpha_{1,2}>$ decompositon for the joint system state

$$|\Psi_T>=(1/\sqrt{2})\sum_{\alpha=1}^{4}\varphi^\alpha_{1,2}\sigma^\alpha_3\varphi_3 \tag{38}$$

we get four decomposition matrixs. $\sigma^1_2=\begin{pmatrix} Y_1 & Y_3 \\ Y_2 & Y_4\end{pmatrix}$, $\sigma^2_2=\begin{pmatrix} Y_1 & -Y_3 \\ Y_2 & -Y_4\end{pmatrix}$, $\sigma^3_2=\begin{pmatrix} Y_3 & Y_1 \\ Y_4 & Y_2\end{pmatrix}$, $\sigma^4_2=\begin{pmatrix} Y_3 & -Y_1 \\ Y_4 & -Y_2\end{pmatrix}$. The collapsed state that Bob can receive is $(1/\sqrt{2})\sigma^\alpha_2$, as long as there exists the inverse matrix of σ^α_2, Bob can obtain the transported state. When $Y^1_2=Y^1_3=Y^2_2=Y^2_3=Y^3_2=Y^3_3=0$, we get the same results as in Ref.[14].

Furthermore , we consider three-qubit case as the example. The collapsed state that Bob can receive is $(1/\sqrt{2})\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8}|\varphi_{4,6,8}>$, as long as there exists the inverse matrix of $\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8}$, Bob can obtain the transported state.

$$(\sigma^{\alpha_1\alpha_2\alpha_3}_{4,6,8})^{-1}=(\sigma^{\alpha_1}_4)^{-1}\otimes(\sigma^{\alpha_2}_6)^{-1}\otimes(\sigma^{\alpha_3}_8)^{-1}, \tag{39}$$

for the quantum channel with parameters $(Y^i_1,Y^i_2,Y^i_3,Y^i_4)$, the inverse matrix of the sub-matrixes are

$$\begin{aligned}(\sigma^1_i)^{-1} &= \frac{1}{Y^i_1Y^i_4-Y^i_2Y^i_3}\begin{pmatrix} Y^i_4 & -Y^i_2 \\ -Y^i_3 & Y^i_1\end{pmatrix},(\sigma^2_i)^{-1}=\frac{1}{Y^i_1Y^i_4-Y^i_2Y^i_3}\begin{pmatrix} Y^i_4 & -Y^i_2 \\ Y^i_3 & -Y^i_1\end{pmatrix},\\ (\sigma^3_i)^{-1} &= \frac{1}{Y^i_1Y^i_4-Y^i_2Y^i_3}\begin{pmatrix} -Y^i_3 & Y^i_1 \\ Y^i_4 & -Y^i_2\end{pmatrix},(\sigma^4_i)^{-1}=\frac{1}{Y^i_1Y^i_4-Y^i_2Y^i_3}\begin{pmatrix} -Y^i_3 & Y^i_1 \\ -Y^i_4 & Y^i_2\end{pmatrix}.\end{aligned} \tag{40}$$

The universal expression of the inverse sub-matrix is

$$(\sigma_i^\mu)^{-1} = \frac{1}{(T_{\mu 1}^i Y_1^i + T_{\mu 2}^i Y_2^i)(T_{\mu 3}^i Y_3^i + T_{\mu 4}^i Y_4^i) - (T_{\mu 3}^i Y_1^i + T_{\mu 4}^i Y_2^i)(T_{\mu 1}^i Y_3^i + T_{\mu 2}^i Y_4^i)} \begin{pmatrix} T_{\mu 3}^i Y_3^i + T_{\mu 4}^i Y_4^i & -T_{\mu 3}^i Y_1^i - T_{\mu 4}^i Y_2^i \\ -T_{\mu 1}^i Y_3^i - T_{\mu 2}^i Y_4^i & T_{\mu 1}^i Y_1^i + T_{\mu 2}^i Y_2^i \end{pmatrix}. \quad (41)$$

Obviously $Y_1^i Y_4^i \neq Y_2^i Y_3^i$ is the necessary condition of the successful teleportation. The existence of the inverse matrix can be used as a criterion to judge if the teleportation is successful. How to realize such an inverse transformation is still an open problem.

(1) Using the decomposition matrix to forecast the collapsed quantum state of the receiver

When the sender Alice measures $|\Psi_T>$ with one of $|\varphi_{1,5}^{\alpha_1}\varphi_{2,7}^{\alpha_2}\varphi_{3,9}^{\alpha_3}>$, the receiver Bob can obtain the collapsed quantum states $(1/\sqrt{2})^3\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}|\varphi_{4,6,8}>$, which are determined by the decomposition matrix.

In Ref.[15], the unknown state and the quantum channel satisfy $Y_2^1 = Y_3^1 = Y_2^2 = Y_3^2 = Y_2^3 = Y_3^3 = 0$, $|\varphi>_{1,2,3} = X_1|000> + X_2|001> + X_3|010> + X_6|100>$, $|\varphi>_{4,5} = Y_1^1|00> + Y_4^1|11>$, $|\varphi>_{6,7} = Y_1^2|00> + Y_4^2|11>$, $|\varphi>_{8,9} = Y_1^3|00> + Y_4^3|11>$. The sub-matrixes of the quantum channel are $\sigma_4^1 = \begin{pmatrix} Y_1^1 & 0 \\ 0 & Y_4^1 \end{pmatrix}$, $\sigma_4^2 = \begin{pmatrix} Y_1^1 & 0 \\ 0 & -Y_4^1 \end{pmatrix}$, $\sigma_4^3 = \begin{pmatrix} 0 & Y_1^1 \\ Y_4^1 & 0 \end{pmatrix}$, $\sigma_4^4 = \begin{pmatrix} 0 & -Y_1^1 \\ Y_4^1 & 0 \end{pmatrix}$, $\sigma_6^1 = \begin{pmatrix} Y_1^2 & 0 \\ 0 & Y_4^3 \end{pmatrix}$, $\sigma_6^2 = \begin{pmatrix} Y_1^2 & 0 \\ 0 & -Y_4^3 \end{pmatrix}$, $\sigma_6^3 = \begin{pmatrix} 0 & Y_1^2 \\ Y_4^2 & 0 \end{pmatrix}$, $\sigma_6^4 = \begin{pmatrix} 0 & -Y_1^2 \\ Y_4^2 & 0 \end{pmatrix}$, $\sigma_8^1 = \begin{pmatrix} Y_1^3 & 0 \\ 0 & Y_4^3 \end{pmatrix}$, $\sigma_8^2 = \begin{pmatrix} Y_1^3 & 0 \\ 0 & -Y_4^3 \end{pmatrix}$, $\sigma_8^3 = \begin{pmatrix} 0 & Y_1^3 \\ Y_4^3 & 0 \end{pmatrix}$, $\sigma_8^4 = \begin{pmatrix} 0 & -Y_1^3 \\ Y_4^3 & 0 \end{pmatrix}$. From Eq.(8) we know that $(1/\sqrt{2})^3\sigma_{4,6,8}^{\alpha_1\alpha_2\alpha_3}|\varphi_{4,6,8}>$ is the collapsed state that Bob can manipulate, taking the sub-matrixes we get the same results as in Ref.[15].

(2) Using the decomposition matrix to determine the manipulation of the receiver

For the maximally entangled channel ($Y_1^i = Y_4^i = 1/\sqrt{2}, Y_2^i = Y_3^i = 0$), $\sigma^1 = \sigma_0/\sqrt{2}$, $\sigma^2 = \sigma_z/\sqrt{2}$, $\sigma^3 = \sigma_x/\sqrt{2}$ and $\sigma^4 = -(i\sigma_y)/\sqrt{2}$, the corresponding inverse matrix are $(\sigma^1)^{-1} = \sqrt{2}\sigma_0$, $(\sigma^2)^{-1} = \sqrt{2}\sigma_z$, $(\sigma^3)^{-1} = \sqrt{2}\sigma_x$ and $(\sigma^4)^{-1} = \sqrt{2}(i\sigma_y)$, where $\sigma_{x,y,z}$ is the Pauli matrix and σ_0 is the identity matrix. Here $(\sigma^\mu)^{-1}$ is the transformation matrix that Bob can manipulate.

(3) Using the decomposition matrix to calculate the probability of successful teleportation

The probability of successful teleportation is determined by the parameters of the quantum channel, while the answer of the GGNM problem is very complex and tedious, we will discuss them later.

5. CONCLUSIONS

In this paper, we propose a method of the Bell bases decomposition to teleport an unknown N-qubit state through a non-maximally entangled quantum channel, and give the general expression of the decomposition matrix for N-qubit case. In theory, we solved the three key problems in GGNM teleportation. Using the decomposition matrix, Bob can easily obtain the collapsed state, the inverse matrix of the decomposition matrix is just the transformation matrix that Bob can manipulate. The sub-matrixes $(\prod_{\alpha_i=1}^{N} \otimes\sigma^{\alpha_i})$ of the decomposition matrix is determined by the parameters of the quantum channel, the inverse matrix of the sub-matrix determines the transformation matrix that Bob can manipulate and its parameters determine the probability of the successful transformation, the existence of the inverse matrix is a criterion that can help us to judge if the teleportation is successful.

There still exists some open problems, for example, how to realize the transformation of the inverse matrixes and how to generalize this method to the quantum channel with smaller assistant qubits.

We would like to thank Pro. X. W. Zha for his warmhearted discussions. This work is supported by the NSF of China under grant 10547008 and the NSF of Shaanxi Province under grant 2004A15.

---

[1] M. A. Nielsen, I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge (2000).

[2] C. H. Bennett, G. Brassard, C. Crepeau *et al.*, *Phys. Rev. Lett.* **70**, 1895 (1993).

[3] D. Bouwmeester, J. W. Pan, K. Mattle *et al.*, *Nature* **390** 575 (1997).

[4] S. B. Zheng and G. C. Guo, *Phys. Rev. A* **63**, 044302 (2001).

[5] E. Solano, C. L. Cesar, R. L. de Matos Filho and N. Zagury, *Eur. Phys. J. D* **13**, 121 (2001).

[6] M. A. Nielsen, E. Knill and R. Laflamme, *Nature* **396**, 52 (1998).

[7] J. Lee, H. Min and S. D. Oh, *Phys. Rev. A* **64**, 014302 (2001); *ibid* **66**, 052318 (2002).
[8] T. Gao, Z. X. Wang and F. L. Yan, *Chin. Phys. Lett.* **20**, 2094 (2003).
[9] C. P. Yang and G. C. Guo, *Chin. Phys. Lett.* **16**, 628 (2000).
[10] J. Fang, Y. Lin, S. Zhu and X. Chen, *Phys. Rev. A* **67**, 014305 (2003).
[11] W. P. Bowen, N. Treps, B. C. Buchler *et al.*, *Phys. Rev. A* **67**, 032302 (2003).
[12] T. J. Johnson, S. D. Bartlett and B. C. Sanders, *Phys. Rev. A* **66**, 042326 (2002).
[13] B. S. Shi, Y. K. Jiang and G. C. Guo, *Phys. Lett. A* **268**, 161 (2000).
[14] W. L. Li, C. F. Li and G. C. Guo, *Phys. Rev. A* **61**, 34301 (2000).
[15] Y. Z. Zheng, L. Y. Dai and G. C. Guo, *Acta. Phys. Sin.* **52**, 2678(in Chinese) (2003).
[16] G. Rigolin, *Phys. Rev. A* **71**, 032303 (2005).
[17] Y. Yeo and W. K. Chua, *Phys. Rev. Lett.* **96**, 060502 (2006).
[18] P. X. Chen, S. Y. Zhu and G. C. Guo, *Phys. Rev. A* **74**, 032324 (2006).